Teachers' Desk Reference:
Practical Information for Pennsylvania's Teachers

Essential Practices for Effective Mathematics Instruction

As a classroom teacher, the instructional practices that are the focus of this *Teachers' Desk Reference* will support you in providing effective mathematics instruction. There are several essential practices that will increase your students' opportunities for success in mathematics:

- Implementing classroom norms
- Developing procedural and conceptual knowledge
- Challenging student misconceptions
- Providing high-level tasks
- Using effective questioning techniques
- Encouraging student discourse
- Performing ongoing assessment

Classroom Norms in a Mathematics Environment

One of your goals as a classroom teacher is to develop a successful, safe, and positive learning experience for your students. Mathematics teachers must understand the importance of their role with regard to encouraging students to maintain a positive attitude toward math content and instruction.

Classroom norms are the behavioral expectations of a classroom. Classroom norms inform learners about how they are expected to treat each other, as well as the materials used for learning. The use of classroom norms in a mathematics environment will enable students to become more proficient and comfortable discussing mathematics concepts and justifying their ideas. This will, in turn, assist them in clarifying and deepening their conceptual understanding, reorganizing and learning alternative ways to solve the same problems, and developing confidence in their ability to think mathematically. As students become comfortable doing mathematics and sharing their ideas with others, they will begin to see themselves as capable of understanding the content and processes inherent in mathematics.

When students take an active role in composing classroom norms, they are more likely to take ownership, participate in instruction, and engage in mutually respectful and cooperative relationships. In addition, students and instructional team members jointly developing and implementing norms shifts some of the responsibility for supporting and encouraging socially-appropriate interactions from the teacher to the students. Once students have demonstrated a basic understanding of the core concepts of trust, sharing, belonging, and respect, the instructional team and the students can jointly develop classroom norms that support the concepts.

Regardless of their level of specificity, classroom norms need to be developed jointly by the instructional team and students. They can be established to develop an expectation that students are to justify their mathematical statements and make them clear to others. Students need to be able to justify and explain ideas in order to make their reasoning clear, hone their reasoning skills, and improve their conceptual understanding.

Just like any other behavioral expectation, classroom norms must be explicitly taught. This will assure the teacher and students that all participants have a clear understanding of what is allowable and expected. An easy way to ensure explicitness is to utilize a T-chart when defining what each classroom norm looks like and sounds like. Teachers should display and refer to the visual when necessary (See Figure 1).

Figure 1. T-Chart Defining Classroom Norms

Looks Like	Sounds Like
Leaning and Looking	"Can you explain your work?"
Listening and Speaking	"I like how you solved that problem."
Facing partners	"Here's another way to look at it."
Looking at each other's work	"I don't understand."

Procedural and Conceptual Knowledge

Learning mathematics well and being able to translate that learning into use outside of the classroom can be difficult for many students. Therefore, it is imperative that teachers have an understanding of the difference between procedural knowledge and conceptual knowledge. Teaching for both types of knowledge will enhance students' learning and facilitate their ability to apply the mathematics they are learning in real-world situations, which is the very purpose of learning mathematics. Mathematics instruction must include both procedural and conceptual knowledge.

Procedural knowledge can be defined as possessing certain skills or knowing what steps are required to complete a task. This helps students understand which applications to use in mathematics and usually is completed in the form of step-by-step instructions. An example might be to teach students how to follow the steps of an algorithm for multiplying a two-digit number by a two-digit number.

Conceptual knowledge can be defined as having a deeper, more meaningful understanding of mathematical relationships and content. It involves not only the use of procedures to assist in completing the task at hand, but thoughtful, reflective learning that extends beyond the rote step-by-step procedural process. An example of teaching for conceptual understanding for multiplying a two-digit number by a two-digit number may include exploring a problem through the use of:

- Manipulatives
- Pictures
- Oral language
- Written symbols
- A real-world situation in which the skill/concept would be used

In order for students to learn mathematics deeply enough to compete in the 21st century, it is necessary that they be provided with opportunities within the classroom to develop both procedural knowledge and conceptual knowledge. This requires teachers to learn new methods for delivering mathematics instruction that involves the use of both procedural and conceptual knowledge. An important factor to remember is that building procedural knowledge and conceptual knowledge is cyclic in that they consistently build upon one another. Procedural knowledge can lead to conceptual knowledge with the right instruction, and students' newfound conceptual knowledge can be transformed into usage for newer, more complex types of procedures in mathematics.

To teach procedural and conceptual knowledge, the teacher can meet the various needs in a classroom by differentiating instruction using various grouping options with whole group, small group, and one-to-one instruction. Teachers intensify elements of explicit and systematic instruction to meet the various individual needs of students.

Student Misconceptions

Each day, educators are challenged to provide effective and efficient mathematics instruction specific to students' strengths and needs. Students often bring misconceptions and erroneous thinking into their daily work. One step toward ensuring quality and effective instruction is to determine what misconceptions students have and to identify the error patterns they make when solving problems. Misconceptions hinder acquisition of new skills.

Teachers must anticipate misconceptions during lesson planning and consider various strategies to provide tangible learning experiences that will clear up misconceptions that may be interfering with student learning. Mathematical tasks should consistently engage teachers at three levels: learner, analyzer, and planner of the activity. Given this understanding, it is critical to be able to address students' misunderstandings before they expand, solidify, and undermine their confidence and understanding.

Eliminating math misconceptions can be difficult. Merely repeating a lesson or providing extra practice won't help. Simply telling the students they are incorrect will probably not correct the issue either. Recognizing student misconceptions and immediately focusing a discussion on them is imperative so that they do not hinder present and future learning. Providing the right guiding questions is critical to discovering misconceptions and building new and deeper conceptual knowledge.

Examples of common student misconceptions:

- "Multiplication always results in a larger number."
- "In fractions, the largest denominator is the largest fraction."
- "A right triangle means the 90 degree angle is on the right side of the shape."
- "Because you add the tops and the bottoms, 2/7 + 3/7 = 5/14."
- "An equal sign means the answer follows."

> "There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is, than the selection or creation of the tasks with which the teacher engages students in studying mathematics."
>
> —Lappan and Briars (1995)

In the resource, *Mathematics Curriculum Topic Study*, guiding questions for student misconceptions can be found to help teachers reflect on their lesson preparation. For example:

- What specific misconceptions or difficulties might a student have about ideas in this topic?
- Are there any suggestions as to what might contribute to students' misconceptions and how to address them?
- Is there an age or grade when students may be more likely to learn certain ideas in the topic?
- What do I need to know to correct and reteach the concepts?

High-Level Tasks

At the heart of teaching well is the core challenge of getting learners engaged in productive work. The use of high-level tasks helps to engage students in core mathematical content. The use of high-level tasks leads to the development of students' implicit ideas about mathematics, how to make sense of it, and how various concepts are needed in order to solve problems.

If we give students well-defined problems in which they simply utilize previously learned procedures, then these tasks will become the expectation—low rigor with little conceptual understanding. When students find themselves in a problem situation that does not have a direct solution path, they may quickly give up and come to believe that they cannot deal with mathematically-ambiguous problems.

- Tasks form the basis for students' opportunities to learn what mathematics is and how one does it.
- Tasks influence learners by directing their attention to particular aspects of content and by specifying ways to process information.
- The level and kind of thinking required by mathematical instructional tasks influences what students learn.

Mathematical tasks become intertwined with the goals, intentions, actions, and interactions of teachers and students. Considerable preparation time is required to set the stage for the students.

Teachers must:

- Anticipate various problem-solution strategies,
- Determine appropriate assessing and advancing questions to help further student thinking, and
- Plan responses to the questions that students are likely to ask.

Teachers can alter the cognitive demand of tasks as needed. This can lead to an effective way of differentiating classroom instruction for students of varying abilities. It can also lead to unnecessarily lowering the cognitive demand of the task if changes are made without strategically planning them.

Students and teachers are both important contributors to how a task is implemented. Although students' levels of cognitive engagement ultimately determine how much learning occurs, the ways and extent to which the teacher supports students' thinking and reasoning is crucial in the ultimate outcome of high-level tasks.

The main reason for focusing on instructional tasks is to influence student learning. If students are given the opportunity to work on challenging tasks in a supportive classroom environment, substantial learning gains in student thinking, reasoning, problem solving, and communication will be observed.

Assessing and Advancing Questions

Purposeful mathematical questions are those that give a teacher access to student understanding and mathematical thinking. Traditional math instruction conditioned students to think that, when the teacher asked for clarification on their answer, their answer was wrong. However, when questioning is used effectively, students realize that the teacher is interested in unveiling their thinking and reasoning in order to move them toward the mathematical goal of a task.

> "Not all tasks are created equal, and different tasks will provoke different levels and kinds of student thinking."
>
> —Stein, Smith, Henningsen, and Silver (2000)

A recommended process for using questioning effectively is as follows (Fisher and Frey, 2007; Marzano, 2001; Driscoll,1999):

- First determine the purpose for the question. Is it an **assessing question** or an **advancing question**? Assessing questions clarify the mathematical task the students have completed and what they understand about the work they have done. An example is, "When it says rule, what does that mean to you?" Advancing questions help students move beyond their current thinking to get them closer to where they need to go. An example is, "What would the next one look like?"
- The next step is to anticipate the responses to the questions and the misconceptions students may have in their learning. The teacher should prepare questioning strategies in advance and be ready to give feedback and reinforcement. Questions should address both procedural knowledge – knowledge of formal language, symbolic representations, rules, algorithms, and procedures – and conceptual knowledge – transferrable knowledge rich in the relationships of mathematical concepts.
- Finally, it is crucial to provide "wait time" or "think time" before students respond. This has the effect of deepening student knowledge. Give students 3 to 5 seconds to allow them to digest the question, retrieve information, and craft a response. This is highly recommended for students who are English Language Learners, who may need time to translate the question mentally.

Instructional time that is designated for cooperative learning should include questioning strategies that help students direct one another and encourage students who are typically reluctant or quiet to be actively engaged in group work. Questioning provides a link to what all students in the classroom understand, as well as tasks with which they may be struggling. It also offers an opportunity for teachers to encourage students to think, articulate, and justify their thinking.

> Teachers provoke students' reasoning about mathematics through the tasks they provide and the questions they ask. Asking questions that reveal students' knowledge about mathematics allows teachers to design instruction that responds to and builds on this knowledge.
> —NCTM (2008)

Student Discourse in a Mathematics Classroom

Supporting meaningful, high quality student interaction in the math classroom is a daunting, yet beneficial strategy. Teachers are often faced with varied student responses to cognitively-demanding tasks and must find ways to use those responses to guide the class toward the lesson's mathematical goal(s). A facilitator's role during class discussions is to develop, and then build on, the individual and group efforts of the students as they wrestle with a task, rather than to simply endorse particular approaches as being correct or demonstrate procedures for solving similar tasks. Teaching practices that emphasize student interaction have been shown to improve problem solving and conceptual understanding without the loss of computational mastery (Bruce, 2007). The benefits increase further when students share their reasoning with one another. Students will generally not engage in high quality math talk without the facilitation of a teacher. Lack of student discourse could drastically diminish the potential learning benefits of completing a high-level task.

Facilitating student discourse is a complex teaching strategy, which takes time and practice to effectively develop. Facilitating student discourse requires planning, a solid mathematical foundation, facilitator skills (so that the teacher is not merely telling and directing), concentrated attention to classroom dynamics, and time (both during a class period and over the course of the academic year). Teachers must decide "what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge" (NCTM, 2008).

Several researchers (Stein, Engle, Smith, and Hughes, 2008) have identified five practices that prove beneficial when facilitating student discourse:

- **Anticipating** likely student responses to cognitively demanding mathematical tasks
- **Monitoring** students' responses to the tasks during the explore phase
- **Selecting** particular students to present their mathematical responses during the discuss-and-summarize phase
- **Purposefully sequencing** the student responses that will be displayed
- **Helping** the class make mathematical connections between different students' responses and between students' responses and the key ideas

Selecting and using cognitively-demanding tasks is important, but not enough. A teacher's actions and reactions impact the quality and amount of student engagement with tasks, as well as students' learning opportunities as presented by engaging in the task. Talk is a tool for promoting an understanding of effective teaching and learning. The facilitator's role becomes vital for ensuring talk about worthwhile and important mathematics, as well as creating the learning environment as students explore, discuss, and make connections through the use of a cognitively-demanding task.

Mathematics Assessment

Assessment is a process used by teachers and students before, during, and after instruction to provide feedback and to adjust ongoing teaching and learning. Effective use of assessment improves student achievement and provide opportunities to appropriately challenge all students at their instructional levels. Assessment is also an important and critical component of the mathematics classroom. While the primary aim of the mathematics classroom is to provide students with successful experiences in order to build and apply mathematical ideas, there is a need to systematically evaluate whether pupil learning has been achieved. Assessment is an integral part of the teaching and learning process.

Formative assessment occurs in the classroom and is a process used by teachers and students during instruction that provides feedback for adjusting ongoing teaching and learning. This will improve students' achievement of intended instructional outcomes. The use of formative assessment will allow educators to emphasize the processes of learning in both the cognitive and affective domains for the purpose of improving teaching and learning (Quek & Fan, 2009; Stiggins, 2007).

Pennsylvania defines formative assessment as classroom-based assessment that allows teachers to monitor and adjust their instructional practice in order to meet the individual needs of their students. Formative assessment can consist of formal instruments or informal observations. The key to formative assessment is how the results are used. Results should be used to shape teaching and learning, but not for grading. Formative assessment encompasses questioning strategies, active engagement check-ins, (such as response cards, white boards, random selection, think-pair-share, and popsicle sticks for open-ended questions), and analysis of student work based on set rubrics and standards, including homework and tests. Assessments are only formative when the information is used to adapt instructional practices to meet individual student needs, as well as providing individual students with corrective feedback that allows them to "reach" set goals and targets. Ongoing formative assessment is an integral part of effective instructional routines. It provides teachers with the information they need to differentiate and make adjustments to instructional practice in order to meet the needs of individual students. Effective teachers seamlessly integrate formative assessment strategies into their daily instructional routines.

Progress monitoring is a type of formative assessment that is used to assess student performance and evaluate the effectiveness of instruction. It guides instruction and determines whether students are responding to instruction and any additional supports and intervention.

Benchmark assessments are designed to provide feedback to both the teacher and the student about how the student is progressing toward demonstrating proficiency on grade-level standards. Well-designed benchmark assessments and standards-based assessments: measure the degree to which a student has mastered a given mathematics concept; measure concepts, skills, and/or applications; reference the standards, not other students' performance; serve as a test to which teachers want to teach; and measure performance regularly, not only at a single moment in time.

Benchmark assessments are repeatable data collection opportunities that assess critical math concepts and procedures. These measures help identify students who may be "at risk" for poor math outcomes and who may need further diagnostic assessment to guide instruction.

When a **diagnostic assessment** is given prior to instruction, the results establish a student's strengths, weaknesses, knowledge, and skills. Beginning with this information allows an educator to remediate students and adjust the curriculum to meet pupils' unique needs. The GMADE, TOMA -2, and KeyMath3 are examples of math diagnostic assessments.

The Pennsylvania Department of Education has developed online classroom diagnostic assessments for Mathematics, Science, and Reading/Literature. These assessments will be available for students in grade 3 through high school.

Summative assessments pursue an overall judgment of progress made at the end of a defined period of instruction. Summative assessments occur at the end of a school level, grade, or course, or are administered at certain grades for purposes of state or local accountability. They are designed to produce clear data on the student's accomplishments at key points in his or her academic career. Scores on these assessments usually become part of the student's permanent record and are statements as to whether or not the student has met, exceeded, or fallen short of the expected standards. The results of these assessments are often reported with reference to standards and individual students. They can be used as diagnostic tools by teachers to plan instruction and guide the leadership team in developing strategies that help improve student achievement. Examples of summative assessments are PSSA and Terra Nova.

The Keystone Exams are end-of-course assessments designed to assess proficiency in the subject areas of Algebra I, Algebra II, and Geometry in the content area of mathematics. These exams are one component of Pennsylvania's new system of high school graduation requirements. Keystone Exams will help school districts guide students toward meeting state standards.

The practices highlighted in this issue of *Teachers' Desk Reference* challenge you to provide effective instruction that will help your students achieve in mathematics.

References:

Bruce, C. D. (2007). Student interactions in the math classroom: Stealing ideas or building understanding. *What Works? Research into Practice*. [Online]. Available: www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Bruce.pdf.

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers grades 6-10.* Portsmouth, NH: Heinemann.

Fisher, D., & Frey, N. (2007). *Checking for understanding: Formative assessment techniques for your classroom.* Alexandria, VA: ASCD

Keeley, P., & Rose, C. (2006). *Mathematics curriculum topic study.* Thousand Oaks, CA: Corwin Press.

Lappan, G., & Briars, D.J. (1995). How should mathematics be taught? In I. Carl (Ed.), *75 years of progress: Prospects for school mathematics* (pp. 131-156). Reston, VA: National Council of Teachers of Mathematics.

Marzano, R. J., Pickering, D. J., & Pollock, J. E. (2001). *Classroom instruction that works: Research-based strategies for increasing student achievement.* Alexandria, VA: ASCD.

National Council for Teachers of Mathematics. (2008). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Quek, K. S., & Fan, L. (2009). Rethinking and researching mathematics assessment in Singapore: The quest for a new paradigm. In K. Y. Wong, P. Y. Lee, B. Kaur, P. Y. Foong, & S. F. Ng (Eds.), *Mathematics education: The Singapore journey* (pp. 411–434). Singapore: World Scientific.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008).Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning.* 10(4), 313-340.

Stein, M.K., Smith, M.S., Henningsen, M., & Silver, E.A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development.* New York: Teachers College Press.

Stiggins, R. (2001). *Student Involved Classroom Assessment* (Third Edition). Columbus, OH: Merrill Publishing.

Commonwealth of Pennsylvania

Tom Wolf

Governor

pennsylvania
DEPARTMENT OF EDUCATION

Bureau of Special Education

Pennsylvania Training and Technical Assistance Network

www.facebook.com/
pattanpublications

rev. 2/15